University of Brighton

Advanced Engineering Centre

6th Sprays SIG workshop
Pollock Halls, University of Edinburgh, 16th August 2019
Sprays in engineering applications: modelling and experimental studies

# Development of a spray simulation tool based on the fully Lagrangian approach

Oyuna Rybdylova, Yuan Li

EPSRC Engineering and Physical Sciences Research Council

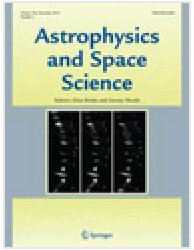
# Outline

- Background
- Basic equations (droplets): original and generalised FLA
- Application of generalised FLA to 1D and 2D flows and preliminary results
- Outlook

University of Brighton

Advanced Engineering Centre

# How to model droplet concentration evolution?




[Astrophysics and Space Science](#)

October 2000, 274:377 | [Cite as](#)

## Lagrangian Modelling of Dust Admixture in Gas Flows

Authors | Authors and affiliations

A.N. Osiptsov 1

1. Institute of Mechanics, Lomonosov Moscow State University, Moscow, Russia

'A generalized Fully Lagrangian Approach for gas-droplet flows' supported by EPSRC (EP/R012024/1)

Dr Oyuna Rybdylova, Dr Timur Zaripov, Dr Yuan Li

# Motivation

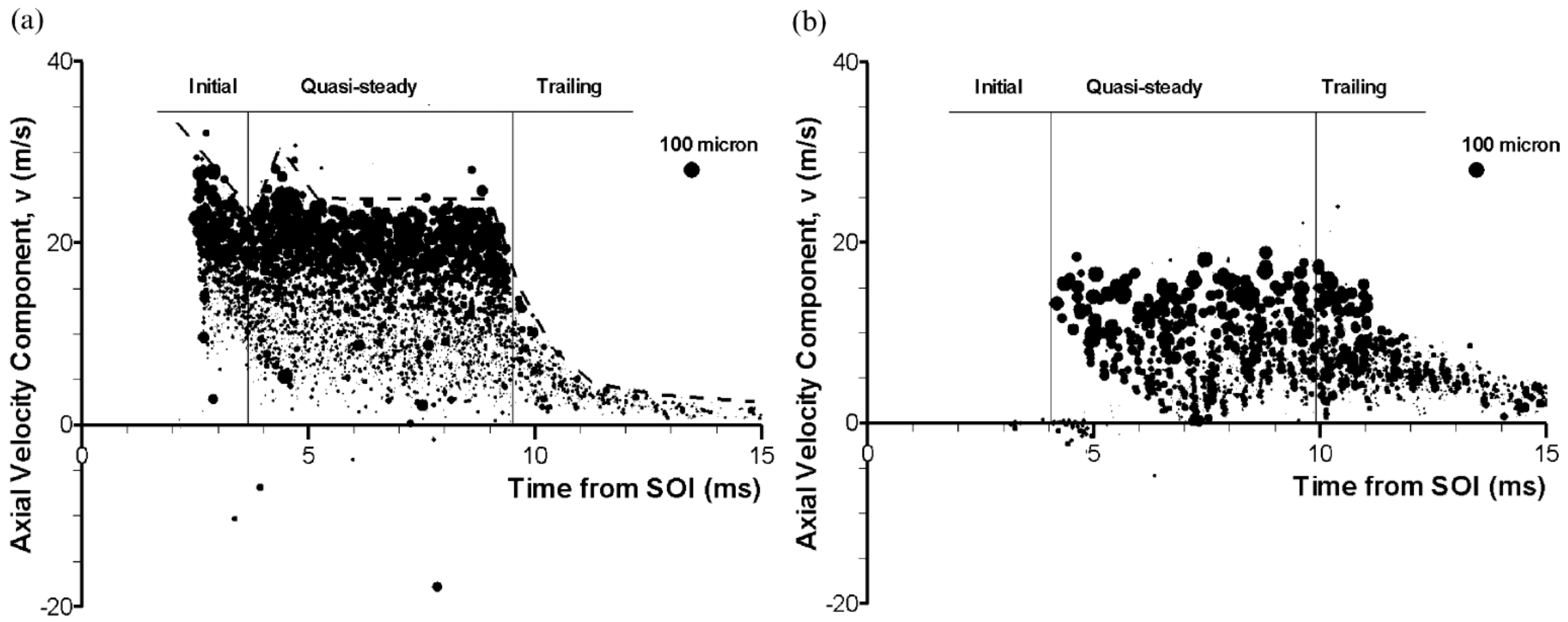



**Fig. 3** Distribution of droplet diameters and velocities in the PFI injector spray plotted against time from SOI when (a) $r = 0$ mm and $x = 15$ mm and (b) when $r = 6$ mm and $x = 55$ mm

S. Begg, F. Kaplanski, S. Sazhin, M. Hindle, M. Heikal. *Int J. Engine Res*. 2009

- Sprays are essentially polydisperse
- Droplet sizes and distribution evolve with time

# Standard FLA

Lagrangian variables are the initial coordinates of the droplet positions: $x_0,\ y_0,\ z_0$

Continuity equation

$$n_d\,|J| = n_{d0},$$

Momentum balance

$$\frac{\partial \mathbf{r}_d}{\partial t} = \mathbf{v}_d, \qquad \frac{\partial \mathbf{v}_d}{\partial t} = \mathbf{f}_d,$$

Energy balance

$$c_{dl}\frac{\partial T_d}{\partial t} = q_d,$$

Equations for Jacobian components

$$\frac{\partial J_{ij}}{\partial t} = q_{ij}, \qquad \frac{\partial q_{ij}}{\partial t} = \frac{\partial f_{id}}{\partial x_{j0}}$$

$$|J| \equiv |\det(J)|$$

$$J_{ij} = \partial x_i / \partial x_{j0}$$

Jacobian of the transformation form Eulerian to Lagrangian coordinates

A system of ODE, initial conditions correspond to the way the dispersed phase is introduced or fed to the flow

# FLA for polydisperse admixture

Lagrangian variables are the initial coordinates of the droplet positions and the initial size:

$$x_0,\ y_0,\ z_0, r_{d0}$$

Continuity equation formulated for the distribution of droplets over space and sizes

$$\tilde{n}_d\left(t, \mathbf{x}, r_d\right)|J| = \tilde{n}_{d0},$$

Jacobian of the transformation form Eulerian to Lagrangian coordinates

$$|J| \equiv |\det(J)|$$

$$J = \begin{pmatrix} J_{11} & J_{12} & J_{13} & J_{14} \\ J_{21} & J_{22} & J_{23} & J_{24} \\ J_{31} & J_{32} & J_{33} & J_{34} \\ J_{41} & J_{42} & J_{43} & J_{44} \end{pmatrix} = \begin{pmatrix} \partial x/\partial x_0 & \partial x/\partial y_0 & \partial x/\partial z_0 & \partial x/\partial r_{d0} \\ \partial y/\partial x_0 & \partial y/\partial y_0 & \partial y/\partial z_0 & \partial y/\partial r_{d0} \\ \partial z/\partial x_0 & \partial z/\partial y_0 & \partial z/\partial z_0 & \partial z/\partial r_{d0} \\ \partial r_d/\partial x_0 & \partial r_d/\partial y_0 & \partial r_d/\partial z_0 & \partial r_d/\partial r_{d0} \end{pmatrix}$$

# FLA for polydisperse admixture

For a chosen particle trajectory, we have the following system of ODE:

$$\frac{\partial \mathbf{x}_d}{\partial t} = \mathbf{v}_d, \quad \frac{\partial \mathbf{v}_d}{\partial t} = \mathbf{f}_d,$$

$$c_{dl}\frac{\partial T_d}{\partial t} = q_d, \quad \frac{\partial r_d}{\partial t} = \dot{r}_d,$$

$$\frac{\partial J_{ij}}{\partial t} = q_{ij}, \quad \frac{\partial q_{ij}}{\partial t} = \frac{\partial f_{di}}{\partial x_k} J_{kj} + \frac{\partial f_{di}}{\partial r_d} J_{4j}, \quad i = 1, 2, 3, \quad j = 1, ..., 4$$

$$\frac{\partial J_{4j}}{\partial t} = \frac{\partial \dot{r}_d}{\partial x_{0j}} J_{4j}, \quad \frac{\partial J_{44}}{\partial t} = \frac{\partial \dot{r}_d}{\partial r_d} J_{44}, \quad i, j = 1, 2, 3.$$

Initial conditions correspond to the way the dispersed phase is introduced or fed to the flow

# 1D flow of droplets in still hot air

Force and heat flux on the droplet:

$$\mathbf{f}_d = 6\pi r_d^* \mu \left(\mathbf{v}^* - \mathbf{v}_d^*\right)$$

$$q_d = 4\pi r_d^* \lambda (T^* - T_d^*)$$

Assume all the heat that reaches the droplet is spent on evaporation:

$$\dot{m} = \frac{q_d}{H}$$

Non-dimensional parameters:

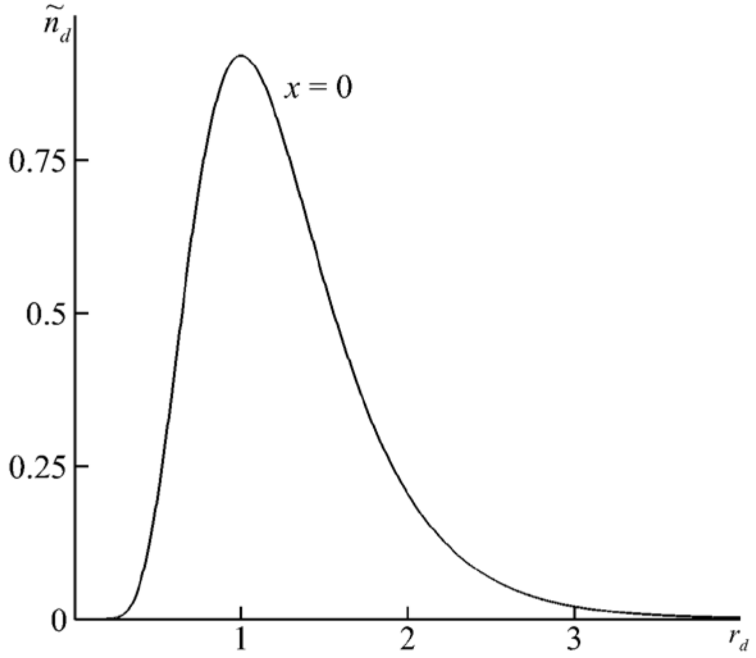
$$x_{(d)} = \frac{x_{(d)}^*}{l_\tau}, \quad u_{(d)} = \frac{u_{(d)}^*}{U}, \quad t = \frac{Ut^*}{l_{\tau 0}}, \quad r_d = \frac{r_d^*}{r_0}, \quad \tilde{n}_d = \frac{\tilde{n}_d^*}{n_{dt}},$$

$$T(T_s) = \frac{T^*(T_s^*) - T_0}{T_a - T_0}. \quad l_{\tau 0} = \frac{m_0 U}{6\pi r_0 \mu}, \quad m_0 = \frac{4}{3}\pi r_0^3 \rho_{dl}$$

Characteristic droplet radius, $r_0$, droplet initial velocity $U$ and temperature $T_0$, $n_{dt}$ total initial droplet number density at $x_0$

# 1D flow of droplets in still hot air

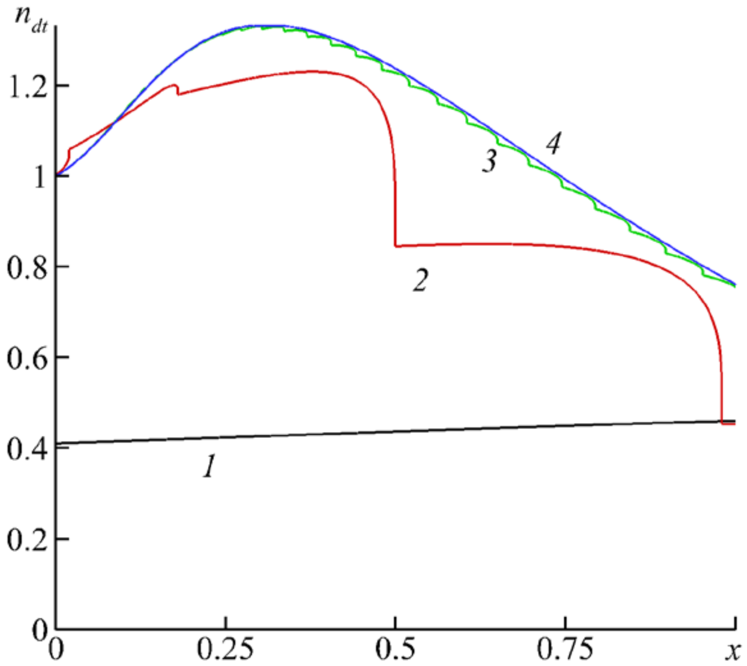
Assume log-normal distribution of droplet sizes at $x_0$ with mean and variance for the corresponding normal distribution $M$ = 0.16 and $S$ = 0.4



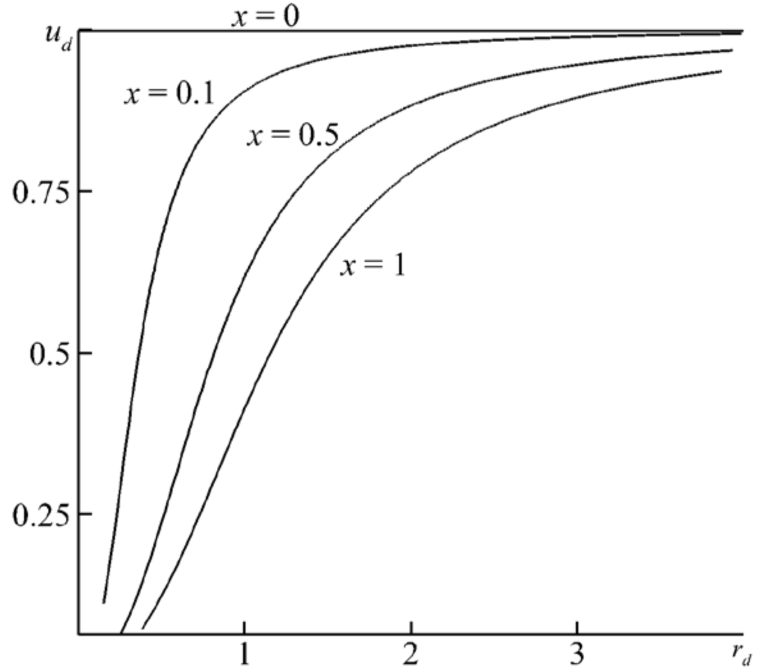

$$\tilde{n}_{d0} = \frac{1}{r_d}\frac{1}{S\sqrt{2\pi}}\exp\left(-\frac{(\ln r_d - M)^2}{2S^2}\right)$$

# 1D flow of droplets in still hot air

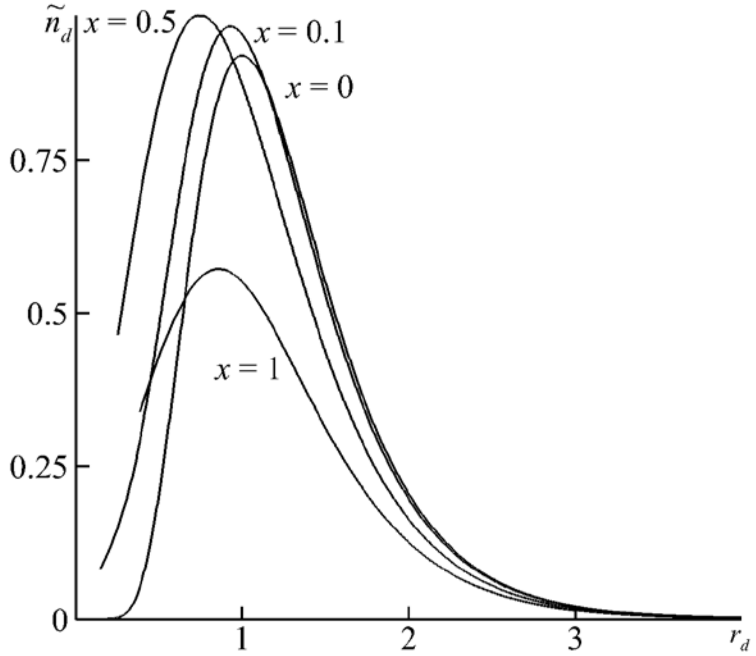
$\delta = 1$




Total number density




Velocity distribution vs size

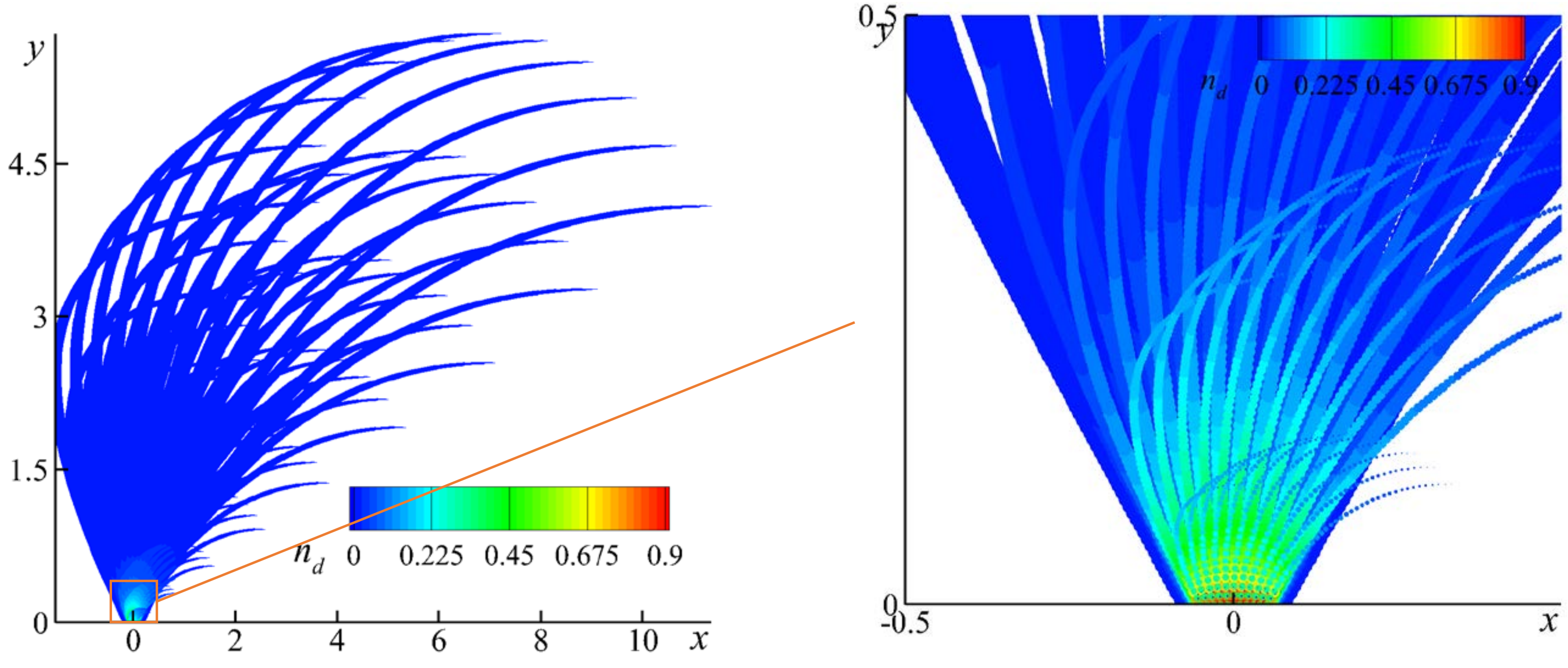



Number density vs size

Discretisation: *1* - 1, *2* - 10, *3* - 100, *4* - 1000

# 2D spray in cross flow

$\delta$ = 1, β = 1

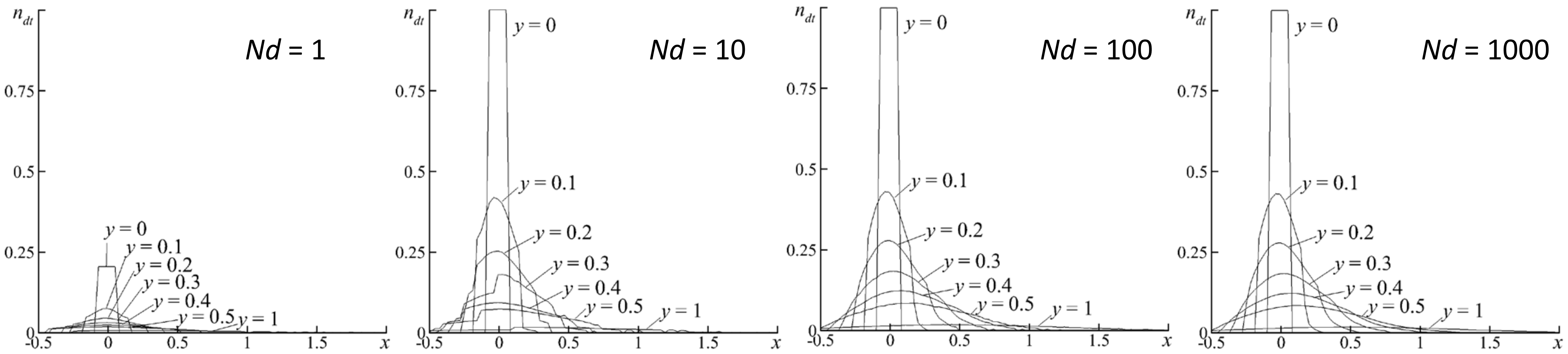



Number density along trajectories

# 2D spray in cross flow

$\delta = 1$, $\beta = 1$

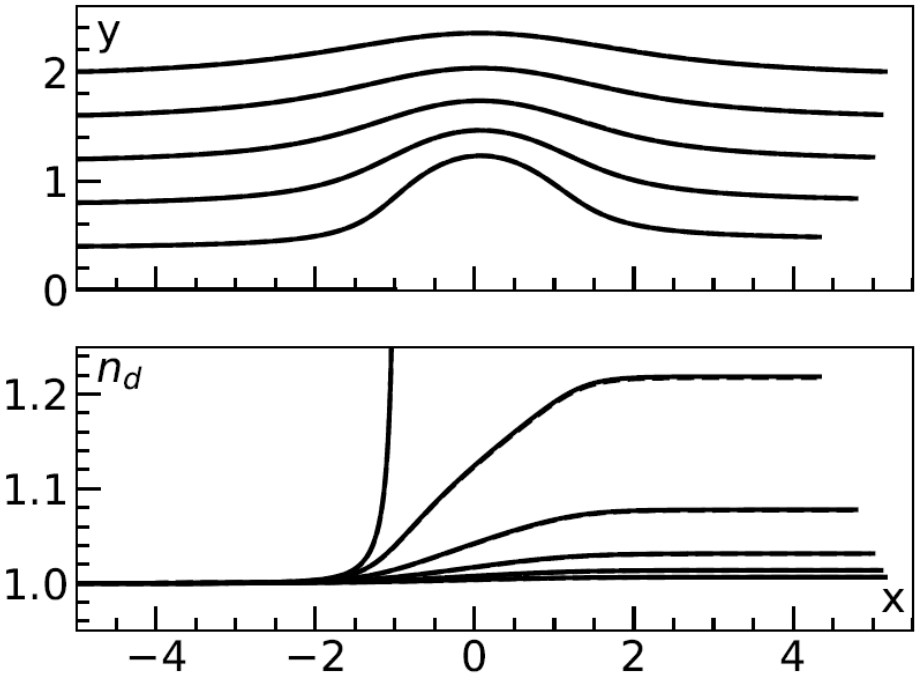
Discretisation:



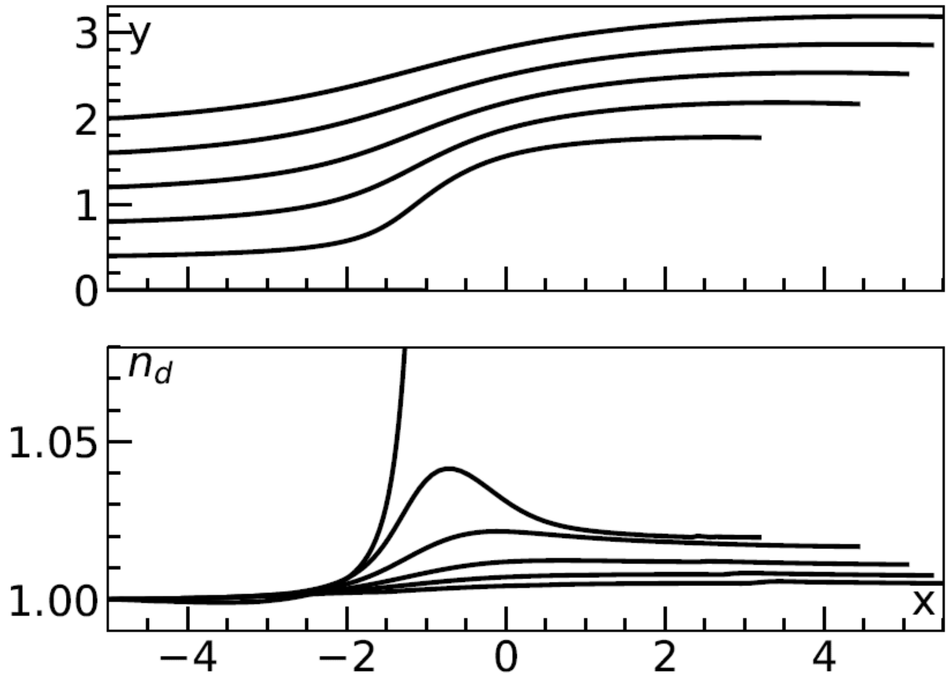

Total number density in horizontal cross-sections

# Flow around a cylinder. OpenFOAM

Steady-state flow, monodisperse particles



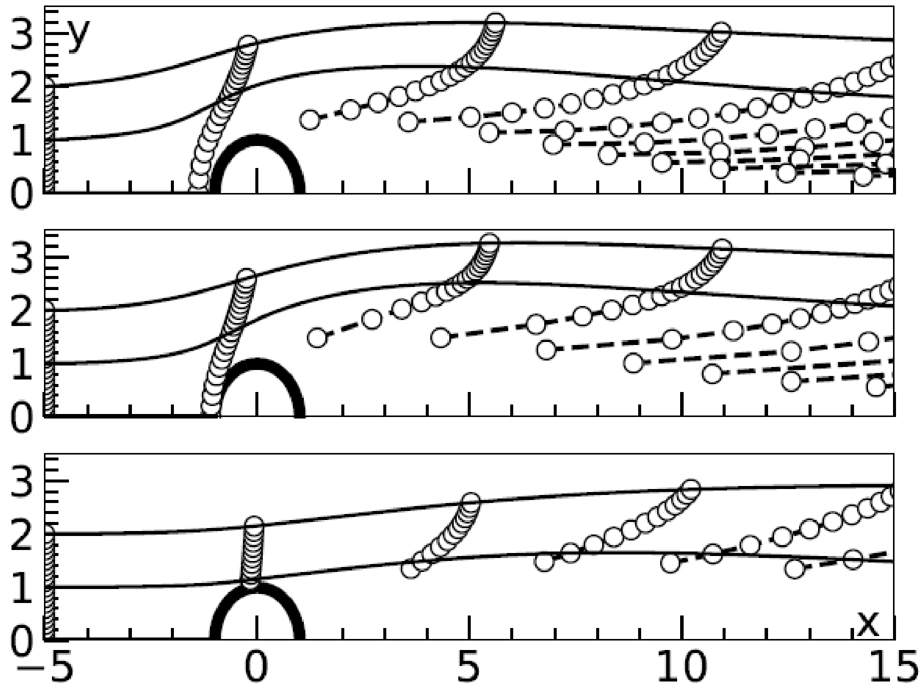

Stk = 0.1 and $\delta$ = 0.1, potential flow




Stk = 0.1 and $\delta$ = 0.1, Re = 40

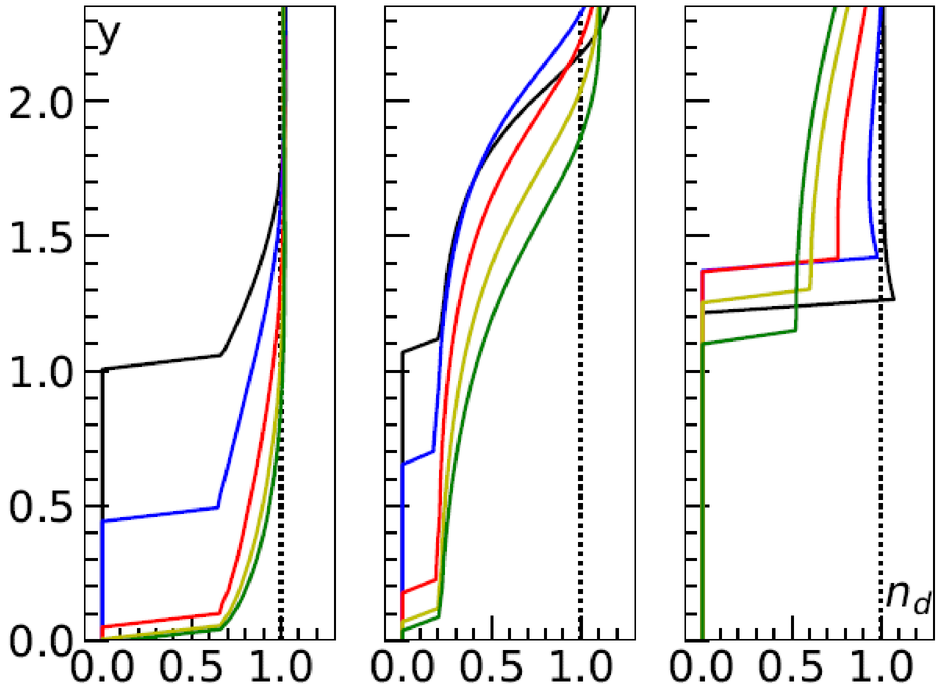
# Flow around a cylinder. OpenFOAM

Steady-state flow, monodisperse particles, Re = 40.




Trajectories and groups

Stk = 0.1 (up), 1 (middle) and 10 (bottom).

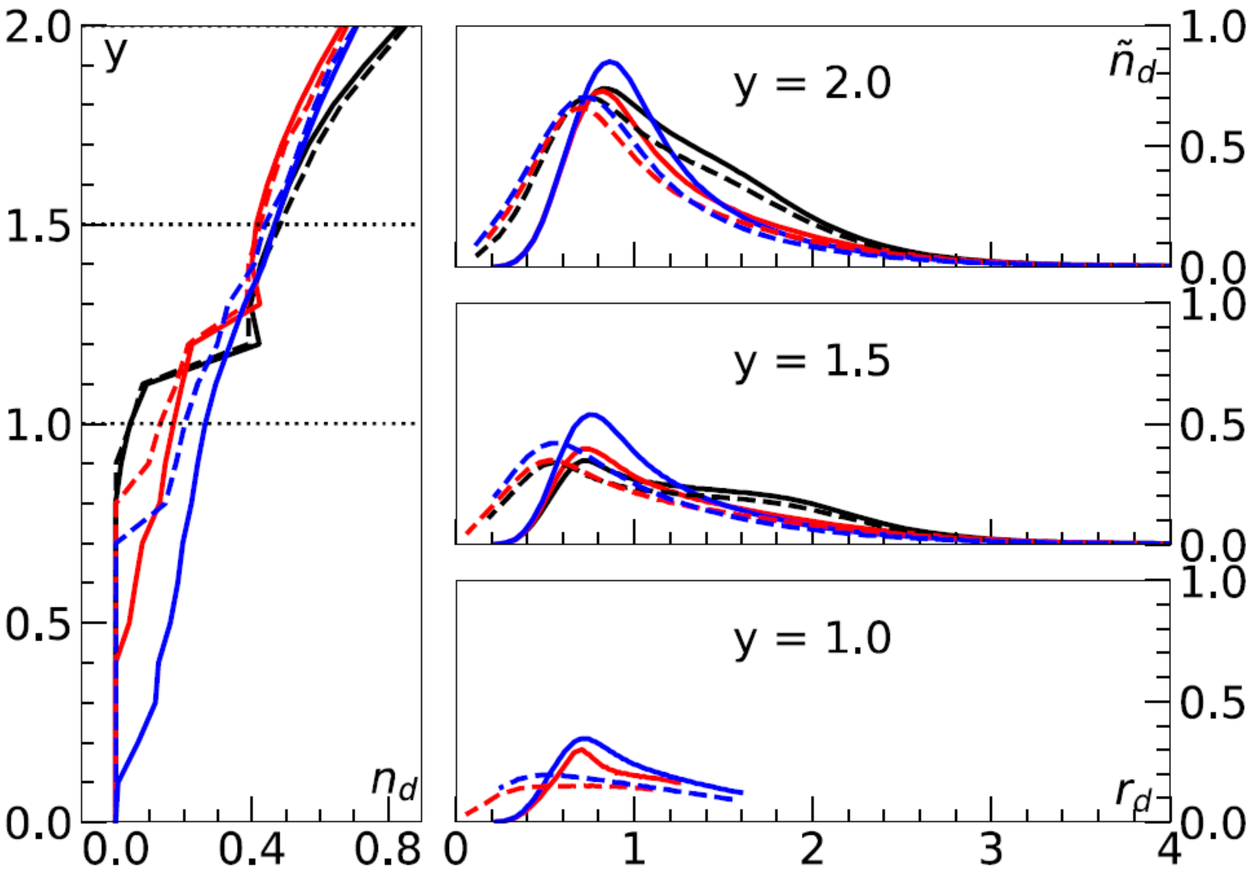



Number density profiles at x = 3 (black), 6 (blue), 9 (red), 12 (yellow) and 15 (green), for Stk = 0.1 (left), 1 (middle) and 10 (right). The dotted line is plotted as a reference line of $n_d = 1$.

# Flow around a cylinder. OpenFOAM

Steady flow, polydisperse particles, Re = 40, reference Stokes number Stk = 1.

Total number density profiles at  x = 3 (black), 6 (red) and 9 (blue).




Distribution of number density against the size of droplets at the chosen points, x = 3 (black), 6 (red) and 9 (blue).

Solid and broken curves stand for the case without and with evaporation ($\delta = 0.05$).

University of Brighton

Advanced Engineering Centre

# Acknowledgements

The authors are grateful to the EPSRC (UK) (grant EP/R012024/1) for their financial support.

University of Brighton

Advanced Engineering Centre

6th Sprays SIG workshop
Pollock Halls, University of Edinburgh, 16th August 2019
Sprays in engineering applications: modelling and experimental studies

# Development of a spray simulation tool based on the fully Lagrangian approach

Oyuna Rybdylova, Yuan Li

EPSRC Engineering and Physical Sciences Research Council